MATH 8
SAMPLE TEST UNIT 4
(6.5, 6.6, CHP 7)

100 POINTS

NAME: _____

Show your work on this paper. EXACT answers are expected unless otherwise specified. No Graphing Calculators. No scratch paper

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These identities will be provided for you on the exam

$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\sin\alpha-\sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

You are expected to know these identities, they will not be provided

Trigonometric Identities

Theorem 11.1. Reciprocal and Quotient Identities: The following relationships hold for all angles θ provided each side of each equation is defined.

- $\sec(\theta) = \dfrac{1}{\cos(\theta)}$
- $\cos(\theta) = \dfrac{1}{\sec(\theta)}$
- $\csc(\theta) = \dfrac{1}{\sin(\theta)}$
- $\sin(\theta) = \dfrac{1}{\csc(\theta)}$
- $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$
- $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$
- $\cot(\theta) = \dfrac{1}{\tan(\theta)}$
- $\tan(\theta) = \dfrac{1}{\cot(\theta)}$

Theorem 11.3. The Pythagorean Identities:

1. $\cos^2(\theta) + \sin^2(\theta) = 1$.
   Common Alternate Forms:
   - $1 - \sin^2(\theta) = \cos^2(\theta)$
   - $1 - \cos^2(\theta) = \sin^2(\theta)$
2. $1 + \tan^2(\theta) = \sec^2(\theta)$, provided $\cos(\theta) \neq 0$.
   Common Alternate Forms:
   - $\sec^2(\theta) - \tan^2(\theta) = 1$
   - $\sec^2(\theta) - 1 = \tan^2(\theta)$
3. $1 + \cot^2(\theta) = \csc^2(\theta)$, provided $\sin(\theta) \neq 0$.
   Common Alternate Forms:
   - $\csc^2(\theta) - \cot^2(\theta) = 1$
   - $\csc^2(\theta) - 1 = \cot^2(\theta)$

Sum and Difference Formulas

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$
$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$
$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$
$$\tan(a+b) = \frac{\tan a+\tan b}{1-\tan a\tan b}$$
$$\tan(a-b) = \frac{\tan a-\tan b}{1+\tan a\tan b}$$

Theorem 11.9. Double Angle Identities: For all applicable angles θ,

- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 2\cos^2(\theta) - 1 \\ 1 - 2\sin^2(\theta) \end{cases}$
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\tan(2\theta) = \dfrac{2\tan(\theta)}{1-\tan^2(\theta)}$

Theorem 11.11. Half Angle Formulas: For all applicable angles θ,

- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos(\theta)}{2}}$
- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos(\theta)}{2}}$
- $\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos(\theta)}{1+\cos(\theta)}}$

where the choice of $\pm$ depends on the quadrant in which the terminal side of $\frac{\theta}{2}$ lies.

Theorem 11.10. Power Reduction Formulas: For all angles θ,

- $\cos^2(\theta) = \dfrac{1+\cos(2\theta)}{2}$
- $\sin^2(\theta) = \dfrac{1-\cos(2\theta)}{2}$

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Fill in the blanks. (2 points each)

(1) Give an identity for $\cos(2\theta)$ = _____

(2) Give an identity for $\sin(\alpha-\beta)$ = _____

(3) Give an identity for $\cos\left(\frac{\theta}{2}\right)$ = _____

(4) $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$ = _____ (exact, simplify)

(5) $\cos 12^\circ \cos 18^\circ - \sin 12^\circ \sin 18^\circ$ =_____ . (exact, simplify)

(6) True or False: $\frac{\sin(4\theta)}{4}$ simplifies to $\sin\theta$_____

(7) Express $\sin 3\theta \cos 7\theta$ as a sum _____

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(8) Using identities, find the exact, simplified value of: (2 points each)
(You must show work, for credit. Calculators should not be used on this problem)

(a) $\tan\left(\frac{-\pi}{12}\right)$_____ (b) $\cos 157.5^\circ$_____

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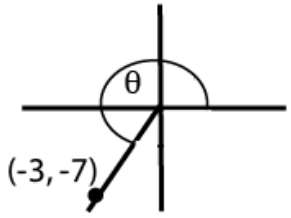
(9) Simplify: $\frac{\tan\theta+\cot\theta}{3\sec\theta\csc\theta}$ (simplifies to a number) (4 points)

(10) Given the following information about θ, (6 points)



Find a) $\cos(2\theta)$ b) $\sin\left(\frac{\theta}{2}\right)$

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(11) Given $\cos\alpha = -\frac{1}{4}$, α in the third quadrant, and $\sin\theta = \frac{5}{13}$, $\frac{\pi}{2} < \theta < \pi$

Find: (6 points)

a) $\cos(\alpha - \beta)$

b) $\tan(2\theta)$

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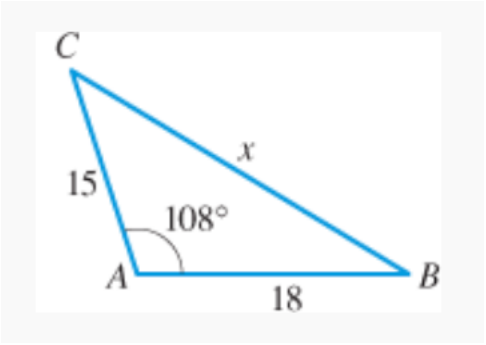
(12) Prove the following identity. Presentation should be very clear. (6 points)

$$1 - \frac{\sin^2\theta}{1+\cos\theta} = \cos\theta$$

(13) Find x, exactly. Then give the approximate value to 3 decimal places: (4 points)



(14). Give the simplified, exact value of the remaining parts for all possible triangles satisfying the given conditions. $\angle A = 30^\circ, \quad a = 6 \quad c = 6\sqrt{3}$ (No need to put values into your calculator for approximations) (6 points)

Find all solutions to the following equations. (6 points each)

(15) $\cos(2x) = 2 + 5\cos x$

(16) $\sin(5x) - \sin(3x) = 0$

SOLVE the following equations: $0 \le x < 2\pi$ (6 points each)

(17) $\sin\theta + 4\sin\left(2\theta\right) = 0$

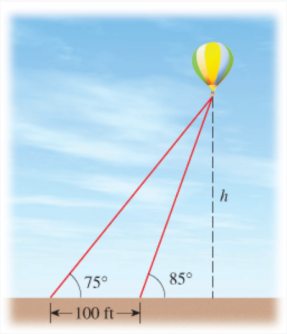
(18) $9 - 4\sin^2\theta = 12\cos\theta$

(19) $4\cos(2\theta) - 4 = 0$

(20) $\sec^2 x - 3\tan^2 x = -5$

(21) (7 points)

Two wires tether a balloon to the ground, as shown. How high is the balloon above the ground?



(22) (7 points)

Length A 100-foot vertical tower is to be erected on the side of a hill that makes a 6° angle with the horizontal (see figure). Find the length of each of the two guy wires that will be anchored 75 feet uphill and downhill from the base of the tower.